1. Index Formula

The USD YLDVOL® index represents the 1-month horizon implied normal volatility under the industry standard Bachelier framework. It is based on a USD 1-month x 10-years at-the-money swaption straddle vs Secured Overnight Financing Rate (from now on, SOFR). Prices are sourced from over-the-counter transactions and the index is updated at 10-minute intervals.

$$\sigma_N = \sqrt{\frac{2\pi}{\tau}} \times \frac{F \times df(T_{1M})}{2 \times \sum_{i=1}^{10} \tau_i \times df\left(T_{pay_i}\right)}$$

Where:

- τ is the year fraction from calculation date to the swaption straddle expiration date (from now on, **Expiration Date**) according to ISDA ACT/360 day count convention.
- F is the USD 1-month x 10-years at-the-money swaption straddle vs SOFR forward premium, expressed in basis points.
- $df(T_{1M})$ is the discount factor from the **Spot Date** to the Effective Date, where:
    - o Spot Date is the day that is two business days after calculation date.
    - o Effective Date is the effective date of the underlying swap (from now on, **Effective Date**) and is set two business days after the Expiration Date.
- τ_i is the year fraction from the i^{th} period accrual start date of the underlying swap (from now, on **Start Date*i***) to the i^{th} period accrual end date of the underlying swap (from now, on **End Date*i***) according to ISDA ACT/360 day count convention.
- $df\left(T_{pay_i}\right)$ is the discount factor from calculation date to the i^{th} period payment date of the underlying swap (from now on, **Payment Date*i*** or T_{pay_i}).

2. Dates

Payment dates of the underlying swap used in the calculation of USD YLDVOL® index are scheduled according to United States (USNY) trading days.

The **Expiration Date** is set one calendar month after the calculation date.

The maturity date of the underlying swap (from now on, **Maturity Date**) is set ten years after the Effective Date. If Effective Date falls on 29th of February, Maturity Date will be the last day of February ten years in the future.

Then, **Payment Date$_{10}$** is equal to Maturity Date and, for i = 9, …, 1, **Payment Date*i*** is set to be (10-i) years prior to the Maturity Date.

Now, apply the ISDA Modified Following business date adjustment convention to Maturity Date and all Payment Date*i*, i = 1, …, 10.

Finally, for each period, i = 1, …, 10, **End Date*i*** is equal to Payment Date*i* and **Start Date*i*** is equal to Payment Date$_{i-1}$, except for **Start Date$_1$** which is equal to the Effective Date.

3. Discount Factors

The 1-month OIS swap is used to calculate the discount factor $df(T_{1M})$.

Let $df(T_{spot})$ be the discount factor from calculation date to the Spot Date:

$$df(T_{spot}) = \frac{1}{(1 + r_{Fed})^{\tau_{spot}}}$$

Where:

- r_{Fed} is the US Federal Funds Effective Rate
- τ_{spot} is the year fraction from the calculation date to the Spot Date, according to ISDA ACT/360 day count convention.

$df(T_{1M})$ is calculated through the par rate balance formula:

$$df(T_{1M}) = \frac{df(T_{spot})}{(1 + r_{1M})^{\tau_{1M}}}$$

Where:

- r_{1M} is the fixed rate coupon of the 1-month OIS swap
- τ_{1M} is year fraction from the Spot Date to the Effective Date according to ISDA ACT/360 day count convention.

To find the risk-free discount factors, a zero-coupon curve based on USD fixed rate vs SOFR swap prices is built. The curve includes OIS swaps with yearly maturities up to 10 years.

Like the 1-month discount factor, the discount factor from Spot Date to the maturity date of the 1-year OIS swap is calculated through the par rate balance formula:

$$df(T_{1Y}) = \frac{df(T_{spot})}{(1 + r_{1Y})^{\tau_{1Y}}}$$

Where:

- r_{1Y} is the fixed rate coupon of the 1-year OIS swap with maturity date equal to T_{1Y}.
- τ_{1Y} is the year fraction from the Spot Date to T_{1Y} according to ISDA ACT/360 day count convention.

For the other yearly maturities, i = 2, …., 10, the corresponding discount factors from Spot Date to the maturity date of the swap T_{iY} are calculated through the bootstrapping technique:

$$df(T_{iY}) = \frac{df(T_{spot}) - r_{iY} \sum_{j=1}^{i-1} \tau_{(j-1)Y,jY} df(T_{jY})}{1 + r_{iY}\tau_{(i-1)Y,iY}}$$

Where:

- r_{iY} is the fixed rate coupon of the OIS swap with maturity date equal to T_{iY}
- $\tau_{(i-1)Y,iY}$ is the year fraction from $T_{(i-1)Y}$ to T_{iY} according to ISDA ACT/360 day count convention.

Finally, zero coupon rates at swap maturities i = 1, …., 10, are calculated as follows:

$$R_{iY} = df(T_{iY})^{-\frac{1}{\tau_{iY}}} - 1$$

Where:

- τ_{iY}, in this case, is the year fraction from the calculation da to T_{iY} according to ISDA ACT/ACT day count convention.

Linear interpolation and extrapolation (applicable only to th last payment date of the underlying swap) on zero coupon rates will be used for the calculation of discount factors $df(T_{pay_i})$ appearing on the USD YLDVOL® index formula.

That is, use R_{iY} and $R_{(i+1)Y}$ to linearly interpolate for the rat r_{pay_i} on swap payment date T_{pay_i} (i = 1, …, 9):

$$r_{pay_i} = R_{iY} + (R_{(i+1)Y} - R_{iY}) \times \frac{T_{pay_i} - T_{iY}}{T_{(i+1)Y} - T_{iY}}$$

Date differences are the number of calendar days between two dates.

Since the final payment date $T_{pay_{10}}$ is beyond T_{10Y} by approximately one month, $r_{pay_{10}}$ is calculated via linear extrapolation:

$$r_{pay_{10}} = R_{9Y} + (R_{10Y} - R_{9Y}) \times \frac{T_{pay_{10}} - T_{9Y}}{T_{10Y} - T_{9Y}}$$

Finally, the discount factors $df(T_{pay_i})$, i = 1, …, 10, are given by:

$$df(T_{pay_i}) = \frac{1}{(1 + r_{pay_i})^{\tau_{pay_i}}}$$

Where τ_{pay_i} is the year fraction from calculation date to swap payment date T_{pay_i} according to ISDA ACT/ACT day count convention.